

15

18

24

25

28

36

42

47

48

49

50  $\sqrt{\frac{50}{88}} * 88$ 

54

54

72

out of  
88due  
after  
"winter"  
breakchapter 3  
in Barron's  
100+  
derivative  
problems

21) Find  $f'(\theta)$  if  $f(\theta) = \sin^2(2\theta^2 - \theta)^3$

$$f(\theta) = [\sin(2\theta^2 - \theta)^3]^2 \quad \swarrow \text{chain rule}$$

$$f'(\theta) = 2[\sin(2\theta^2 - \theta)^3] \frac{d}{dx} [\sin(2\theta^2 - \theta)^3] \quad \swarrow \text{chain rule}$$

$$= 2[\sin(2\theta^2 - \theta)^3] [\cos(2\theta^2 - \theta)^3] \frac{d}{dx} (2\theta^2 - \theta)^3 \quad \swarrow \text{chain rule}$$

$$= 2[\sin(2\theta^2 - \theta)^3] [\cos(2\theta^2 - \theta)^3] [3(2\theta^2 - \theta)^2] \frac{d}{dx} (2\theta^2 - \theta) \quad \swarrow \text{chain rule}$$

$$= 2[\sin(2\theta^2 - \theta)^3] [\cos(2\theta^2 - \theta)^3] [3(2\theta^2 - \theta)^2] [4\theta - 1]$$

13) Given That  $f(0)=4$  and  $f'(0)=-1$   
find an equation of the tangent line to the  
graph of  $y=f(x)$  at the point where  $x=0$ .

to write the equation of a line:

$$\text{slope} = f'(0)$$

$$\text{slope} = -1$$

$$Pt = (0, f(0))$$

$$Pt = (0, 4)$$

$$y - 4 = -1(x - 0)$$

$$y - y_0 = m(x - x_0)$$

$$\begin{aligned} y &= mx + b \\ y &= -x + b \\ 4 &= -(0) + b \\ 4 &= b \\ y &= -x + 4 \end{aligned}$$

(14)

$$f(x) = x^2 + 1$$

a) Average rate of change =

$$\text{SLOPE} = \text{difference quotient} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(-1) - f(-2)}{(-1) - (-2)} = \frac{[(-1)^2 + 1] - [(-2)^2 + 1]}{(-1) - (-2)}$$

$$= \frac{[2] - [5]}{+1} = -3$$

b) instantaneous rate of change = slope of tangent line = value of derivative

$$f(x) = x^2 + 1$$

$$f'(x) = 2x \quad \text{AT } x = -2$$

$$f'(-2) = 2(-2) = -4$$

NO  
LIMIT  
!

$$15) f(x) = \begin{cases} x^2 - 5 & x \leq 1 \\ x - 5 & x > 1 \end{cases}$$

$$f(x) = 2x \text{ when } x \geq 1$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

hmmm...

$$f'(x) = \begin{cases} 2x & x < 1 \\ 1 & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} 2x = 2$$

$$\lim_{x \rightarrow 1^+} f'(x) = \lim_{x \rightarrow 1^+} 1 = 1$$

 $\therefore$  no derivative at 1 $f(x)$  is continuous1)  $f(x)$  is defined at  $x=1$ 

$$2) \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 5 = -4$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} x - 5 = -4$$

 $\therefore$  two-sided limit exists

$$3) f(1) = \lim_{x \rightarrow 1} f(x)$$

 $\therefore f(x)$  is continuous at  $x=1$ 

4) [and everywhere else]

